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On tandem stochastic networks with time-deteriorating product quality

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We consider an *n*-site tandem stochastic production network where each product moves sequentially through the sites, and the product's quality deteriorates with its sojourn time in the system. At each site the product goes through two stages: the first stage is a processing operation with a generally-distributed random duration. This operation either does or does not conclude successfully; in the latter case, the operation is repeated immediately. Once the processing operation concludes successfully, the product goes through an inspection stage lasting a generally-distributed random duration. At the end of the inspection the product's state is determined as follows: either (i) it requires additional processing and moves forward to the next site; or (ii) it is found 'good' and exits the network with quality value depending on its total sojourn time in the system; or (iii) it is declared 'failed', discarded, and exits the network with zero quality value. Two scenarios are analysed: (i) a new product enters the system only after the preceding product has exited and (ii) the network is a tandem Jackson-type system. For each scenario, we construct both time-dependent and quality-dependent performance measures. In the case where the sites can be arranged in an arbitrary order, we derive easy to implement optimal index-type policies of ordering the sites so as to maximise the quality rate of the production network.

Keywords: tandem stochastic networks; product quality deterioration; optimal index-type ordering policies; Jackson networks

1. Introduction

In numerous goods-producing industries, the quality of products deteriorates during the production process. Such industries include the fresh food industry (Bortolini et al. 2016), the metal processing industry (Naebulharam and Zhang 2013), and the polymer manufacturing industry (Colledani et al. 2015; Colledani, Horvath, and Angius 2015), among many others. In such industries, the design of the production system can substantially affect the total time the product resides in the system – and, accordingly, the final quality of the product (see, e.g. a survey by Inman et al. 2013).

One of several dimensions of production system design is the order of the various operations required to produce a product (see, e.g. Bortolini et al. 2017; Perlman 2013). Some production environments are flexible, allowing the final product to be produced according to several different orders of operations (see, e.g. Zhang and Yue 2011). Many studies on flexible manufacturing systems have examined how system flexibility influences productivity and cost (see De Toni and Tonchia 1998; Buzacott and Shanthikumar 1993), but few have considered how the order of operations might influence product quality, despite the prevalence of products that deteriorate over the course of the production process. The current study further addresses this issue.

A manufacturing flow line can be modelled as a tandem stochastic network with reliable servers or unreliable servers, where the buffers (storage areas) between servers have either finite or infinite capacity (Dallery and Frein 1993; Avinadav and Perlman 2013; Perlman, Elalouf, and Bodinger 2014). In this paper, we study an *n*-site tandem stochastic network, where each site can be considered as a reliable processing machine. Processing durations as well as inspection durations are assumed to be variable, and, accordingly, the time period that a product spends at a given site is random. In addition, it is assumed that, at each site, the processing operation either concludes successfully or does not conclude successfully; in the latter case, the operation must be immediately repeated until success is achieved (see, e.g. Yechiali 1988; Brandon and Yechiali 1991). Once a product successfully completes its processing at site i, it is inspected and its state is determined according to one of three possibilities: (i) the product requires additional treatment and it advances to site $i+1$; (ii) the product is declared as 'good' (i.e. acceptable) and it exits the network with a quality value that depends on its total sojourn

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time in the network; and (iii) the product is declared as a 'failed' one, discarded and exits the network with zero quality value.

In what follows (Section 3), we develop a model for the system described, analysing two scenarios: a scenario in which a product can enter the network only after the preceding product exits (i.e. each product traverses the network alone), and a scenario in which multiple products can queue at each site. Next, considering the case where the network is flexible (i.e. that production sites can be arranged in any order) and a product must pass through all sites during production, we seek to optimise the order of sites so that the network's quality rate is maximised. It turns out that it is possible to construct easy to implement index-based ordering policies for the first scenario described above.

2. Literature review

Our work is motivated by various works in the literature that consider the effects of deteriorating product quality in production systems (see a review in Naebulharam and Zhang 2014). The classical economic order quantity (EOQ) and economic production quantity (EPQ) models and their extensions were among the first to incorporate product deterioration in the evaluation of production processes (Goyal and Giri 2001). Another set of studies considers product deterioration while optimising jointly maintenance and inventory decisions (see a review by Van Horenbeek et al. (2013); Perlman, Mehrez, and Kaspi (2001); Perlman and Kaspi (2007) Rivera-Gómez, Gharbi, and Kenné (2013) for representative results). Yet, studies in this vein model the production system as a single (rather than a multi-site) entity and do not address quality deterioration over the course of the production process; rather, products are assumed to deteriorate only after production is complete (e.g. while being held in inventory).

Quality deterioration is also addressed in the literature on queueing systems. In these models, quality deterioration may be represented by impatient customers (see Brandt and Brandt 1999; Movaghar 2006; Altman and Yechiali 2006, 2008 for representative results). However, studies in this area tend to focus on single-stage queueing systems with a single server or with parallel servers, and thus can provide only limited insight regarding quality evaluation and optimisation in a multi-stage production process. In a different setting, a single-processor scheduling problem with deteriorating jobs was studied in Browne and Yechiali (1990) where a fixed number of N jobs are to be processed on a single machine. While waiting for processing jobs deteriorate, causing the random processing-time requirement of each job to grow linearly at a job-specific rate. Optimal scheduling policies that minimise the expected makespan were derived and applications to inventory issuing policies were discussed.

The probabilistic study of networks of queues has been heavily influenced by the works of R.R.P. Jackson (1954, 1956) and J.R. Jackson (1957, 1963). The Markovian models developed in these studies – so-called Jackson networks – are characterised by a product-form solution. Specifically, the multi-dimensional joint probability distribution function of the site occupancies is given as the product of the occupancy distribution function at each site, as if the sites are not connected. Recently several studies addressed the issue of quality deterioration in tandem stochastic networks. Shi and Gershwin (2012) studied a two-machine line in which the waiting time of a part between operations should not exceed a given value in order to guarantee the quality of the completed part. Naebulharam and Zhang (2014) studied Bernoulli reliability serial Markovian lines with deteriorating product quality, where a product's quality depends on its total sojourn time in the buffer. Colledani, Angius, and Horváth (2014) analysed similar manufacturing systems under lead-time dependent product deterioration. For two machine lines with Markovian machines, the authors proposed an approach based on the analysis of the system's steady-state probabilities and on the derivation of the time to absorption. In a related work, Colledani et al. (2015) developed a modelling framework to predict the distribution of production lead times in a multi-stage manufacturing system. Products were assumed to deteriorate over the course of the production process, such that the production lead-time distribution could be used to compute the system's performance in terms of production quality. Bouslah, Gharbi, and Pellerin (2018) developed a model that jointly optimises the production, quality and maintenance control settings for a two-machine production line in which production quality degrades over time. Our work studies the problem from a different angle by considering the optimisation of a flexible tandem network, in which the order of operations can be controlled so as to maximise the system quality rate.

Related stream of research is the design of in-process buffers in industries producing perishable or deteriorating products. Liberopoulos and Tsarouhas (2002) showed that installation of in-process buffer at a specific point of the line led to a reduction in failure impact on product quality and an increase of the system's efficiency. In Liberopoulos, Kozanidis, and Tsarouhas (2007), the authors focused on the production rate of asynchronous production lines in which machines are subject to failures. If the failure time of a machine is long enough, the material being processed in the upstream machines must be scrapped by the system. In Wang, Hu, and Li (2010), a transient analysis was proposed to design the size of the buffers needed in dairy filling and packaging lines. Recently, Angius, Colledani, and Horvath (2018) studied a production control policy for unreliable manufacturing systems that aims at maximising the throughput of parts that satisfy a given lead time constraint. The authors proposed a state-dependent policy that considers the actual level of the buffer. Our work studies the installation of in-process buffers from a different perspective. Specifically, we further investigate the effect of buffers on the system's quality rate.

3. The model

Consider an n-site tandem stochastic network (TSN), where products move sequentially from site to site, passing 2 stages at each site, as depicted in Figure 1.

A single processing duration at site i is a generally distributed random variable B_i, with finite mean $E[B_i]$, finite second moment, and Laplace-Stieltjes transform (LST) $\tilde{B}_i(\theta) \equiv E[e^{-\theta B_i}]$, $i = 1, 2, \ldots, n$. A single processing attempt either concludes successfully, with probability s_i, or does not conclude successfully, with the complementary probability $1 - s_i$; in the latter case the operation is immediately repeated, with the same probability of success. Thus, the total processing time of a single product at site i, G_i, is a Geometric sum of iid random variables (each one distributed like B_i) with mean $E[G_i] = E[B_i]/s_i$ and LST given by $\tilde{G}_i(\theta) \equiv s_i\tilde{B}_i(\theta)/1 - (1 - s_i)\tilde{B}_i(\theta)$.

Once a product's processing stage concludes successfully at site i, the product moves to the second stage to be inspected. The inspection time is a generally distributed random variable, D_i, with mean $E[D_i]$ and LST $\tilde{D}_i(\theta)$. Upon concluding the inspection, the state of the product is determined according to one of three possibilities: (i) with probability a_i the product requires additional processing and it advances to site $i+1$; (ii) with probability p_i the product is declared to be 'good', and it exits the network with a quality value Q_i, depending on the total time it has sojourned in the network, where it is assumed that the quality of the product deteriorates exponentially with time; and (iii) with probability f_i the product is declared as 'failed', discarded and exits the network with a zero quality value.

When processing and inspection concludes at the last site, site n, the product is either 'good' or 'failed'. Thus, for $i = 1, 2, \ldots, n-1$ we have $p_i + a_i + f_i = 1$, whereas for $i = n$, $p_n + f_n = 1$. For example, if $p_i = 0$ for $i = 1, 2, \ldots, n-1$, that is, the product must pass through all sites, then a good product can be realised only with probability $\left(\prod_{i=1}^{n-1} a_i\right) p_n$.

Quality of products that deteriorate as a function of time is usually modelled in the literature as a random variable subject to e.g. Exponential, Gamma or Weibull distribution function (see a survey of Raafat (1991) and a recent review in Naebulharam (2014)). In practice, the quality of food products as a function of time may follow an exponential quality decay function depending on the actual process that is driving the decay. For example, when quality degradation depends on microbial growth (e.g. fresh meat and fish), the quality degradation follows an exponential quality decay (see, e.g. Rong, Akkerman, and Grunow 2011). More examples of exponential quality deterioration functions as a function of the residence time of a product in a buffer are given in Naebulharam (2014).

Let T_j denote a product's accumulated traversal time from site 1 to j. The quality of a product exiting the network at site j, is either $Q_j = \delta_j e^{-\gamma T_j}$ with probability p_j, or zero with probability f_j, where δ_j is the potential quality in case $T_j = 0$, while γ is a positive constant that serves as a scaling factor.

We consider two scenarios. Under *Scenario 1* the network is a sequential tandem configuration in which each single product moves from site to site, with no delay between sites, until it exits the network. A real-life example for this scenario is a situation when there is only one server that performs all the production and inspection stages, so that a new product can enter the production process only when the server becomes available. Another example is a case with an ample number of TSN networks so that each product traverses alone in its isolated network. From a theoretical point of view, under this scenario closed form expressions for the expected quality and for the expected quality rate can be derived for a generally distributed processing and inspection times. *Scenario 2* is the frequently applied tandem Jackson network (TJN) (see, e.g. Yechiali 1988; Brandon and Yechiali 1991) where each processing stage and each inspection stage is an M/M/1-type queue. This scenario allows us to model cases where in-process buffers are installed in transfer lines to increase the effective throughput. In a real-life case study, to improve the performance of food production lines, Liberopoulos and Tsarouhas (2002) present a cost-effective way of speeding up a croissant-processing line by inserting an in-process buffer-refrigerator in the middle of the line. Thus, in the current work, we further investigate whether inserting buffers can improve the system's quality rate.

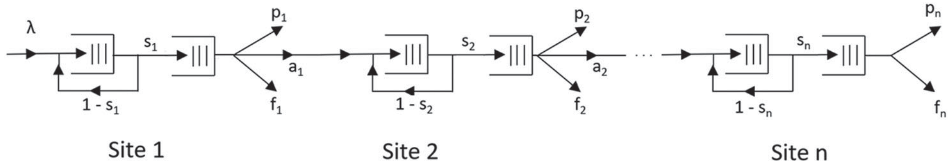


Figure 1. Tandem stochastic network.

3.1. Scenario 1: each product traverses the network alone

Let R_i denote the sum of processing and inspection times of a product at site i. That is, $R_i = G_i + D_i$. Let T_j denote the accumulated traversal time from site 1 to j, i.e. $T_j = \Sigma_{i=1}^{j} R_i$. As the sites are independent, the LST of T_j is $\tilde{T}_j(\theta) \equiv \prod_{i=1}^{j} \tilde{R}_i(\theta) = \prod_{i=1}^{j} \tilde{G}_i(\theta)\tilde{D}_i(\theta)$ and its mean is $E[T_j] = \sum_{i=1}^{j} E[R_i] = \sum_{i=1}^{j} (E[G_i] + E[D_i]))$.

Let T denote the total time a product sojourns in the network before exiting (either as 'good' or as 'failed'). Then,

$$T = T_j \quad \text{w.p.} \left(\prod_{k=1}^{j-1} a_k\right)(p_j + f_j), j = 1, 2, \ldots, n, \quad \text{where} \prod_{k=1}^{0} a_k = 1. \tag{1}$$

Thus, the LST of T is given by

$$\tilde{T}(\theta) = \sum_{j=1}^{n} \left(\left(\prod_{k=1}^{j-1} a_k\right)(p_j + f_j) \prod_{i=1}^{j} \tilde{R}_i(\theta) \right), \tag{2}$$

and its mean is

$$E[T] = \sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} a_k\right)(p_j + f_j) \sum_{i=1}^{j} E[R_i] \tag{3}$$

The expected sojourn time of a product exiting the network as 'good' is

$$E[T|good] = \frac{\sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} a_k\right) p_j \sum_{i=1}^{j} E[R_i]}{\sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} a_k\right) p_j} \tag{4}$$

The quality of a product exiting the network after completing its processing and inspection at site j, is either $Q_j = \delta_j e^{-\gamma T_j}$ with probability p_j, or zero with probability f_j, where δ_j is the potential quality in case $T_j = 0$, while γ is a positive constant that serves as a scaling factor. It then follows that

$$E[Q_j] = E\left[\delta_j e^{-\gamma(\sum_{i=1}^{j} R_i)}\right] = \delta_j \prod_{i=1}^{j} \tilde{R}_i(\gamma) \tag{5}$$

Let Q denote the quality of a product exiting the network (either as good or as failed). Then,

Proposition 1 Under Scenario 1:

The expected quality of a product exiting the network is $E[Q] = \sum_{j=1}^{n} \left(\left(\prod_{k=1}^{j-1} a_k\right) p_j \delta_j \prod_{i=1}^{j} \left(\frac{s_i \tilde{B}_i(\gamma)\tilde{D}_i(\gamma)}{1-(1-s_i)\tilde{B}_i(\gamma)} \right) \right)$

The expected quality rate of the production network is $E[Q]/E[T]$.

Proof Since a product exits the network either as 'good' or as 'failed', we have

$$\begin{aligned} E[Q] &= \sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} a_k\right) p_j E[Q_j] + \sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} a_k\right) f_j \cdot 0 \\ &= \sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} a_k\right) p_j \delta_j \prod_{i=1}^{j} \tilde{R}_i(\gamma) = \sum_{j=1}^{n} \left(\sum_{k=1}^{j-1} a_k\right) p_j \delta_j \prod_{i=1}^{j} \left(\frac{s_i \tilde{B}_i(\gamma)\tilde{D}_i(\gamma)}{1-(1-s_i)\tilde{B}_i(\gamma)} \right). \end{aligned}$$

In the case where products travers the network one by one, their exit rate, i.e. the throughput, is $1/E[T]$. Thus, the expected quality rate of products exiting the system is $E[Q]/E[T]$. ■

3.2. Scenario 2: products queue between sites

Suppose now that the set of processing sites together with the set of inspection sites comprise a tandem Jackson network (TJN), where each stage is an M/M/1-type queue with unlimited buffer. Consider first the processing stage. Assuming that a single processing attempt at site i, B_i, is exponentially distributed with mean $1/\mu_i$, it then follows that the total duration of the processing stage, G_i, is exponentially distributed with parameter $s_i\mu_i$, LST $\tilde{G}_i(\theta) = s_i\mu_i/(s_i\mu_i + \theta)$ and mean $E[G_i] = 1/s_i\mu_i$. The arrival rate to processing stage of site i is Poisson with intensity $\lambda_i = \lambda \prod_{k=1}^{i-1} a_k$. It follows that the total sojourn time, W_i, of a product at the processing stage of site i is exponentially distributed with parameter $s_i\mu_i - \lambda_i$ (see Karlin 1966, 433) with mean $E[W_i] = 1/s_i\mu_i - \lambda_i$ and LST $\tilde{W}_i(\theta) \equiv (s_i\mu_i - \lambda_i)/(s_i\mu_i - \lambda_i + \theta)$.

Similarly, the inspection stage at site i is also an M/M/1 queue, with service rate ξ_i and arrival rate λ_i. Thus, the sojourn time V_i of a product there is also exponentially distributed with parameter $\xi_i - \lambda_i$ and LST $\tilde{V}_i(\theta) \equiv (\xi_i - \lambda_i)/(\xi_i - \lambda_i + \theta)$.

The total traversal time of a product from site 1 to j is $T_j^{TJN} = \sum_{i=1}^{j} (W_i + V_i)$ and its mean is $E[T_j^{TJN}] = \sum_{i=1}^{j} (1/(s_i\mu_i - \lambda_i) + 1/(\xi_i - \lambda_i))$. Similarly to the derivations of $E[T]$ and $E[T|good]$ for scenario 1, the corresponding measures, $E[T^{TJN}]$ and $E[T^{TJN}|good]$ are given, respectively, by Equations (3) and (4), where $(1/(s_i\mu_i - \lambda_i) + 1/(\xi_i - \lambda_i))$ replaces $E[R_i]$ in both formulas. Since $\lambda_i = \lambda \prod_{k=1}^{i-1} a_k$, steady-state is achieved if and only if $\lambda_i < \min\{s_i\mu_i, \xi_i\}$ for $i = 1, \ldots, n$.

As in scenario 1, the quality of a product exiting the network after completing its processing and inspection stages at site j is either $Q_j^{TJN} = \delta_j e^{-\gamma\, T_j^{TJN}}$ with probability p_j, or zero with probability f_j. We claim the following:

Proposition 2 Under Scenario 2:

The expected quality of a product exiting the network is

$$E[Q^{TJN}] = \sum_{j=1}^{n} \left(\left(\prod_{k=1}^{j-1} a_k \right) p_j \delta_j \prod_{i=1}^{j} \left(\frac{s_i\mu_i - \lambda_i}{s_i\mu_i - \lambda_i + \gamma} \cdot \frac{\xi_i - \lambda_i}{\xi_i - \lambda_i + \gamma} \right) \right)$$

The expected quality rate of the production network is $\lambda E[Q^{TJN}]$.

Proof Replacing $\tilde{R}_i(\gamma)$ in Proposition 1 by $(s_i\mu_i - \lambda_i)/(s_i\mu_i - \lambda_i + \gamma) \cdot (\xi_i - \lambda_i)/(\xi_i - \lambda_i + \gamma)$ completes the proof of (i). In a steady-state network, the exit rate equals the arrival rate, λ. Thus, the expected quality per unit of time given in (ii) is $\lambda E[Q^{TJN}]$. ∎

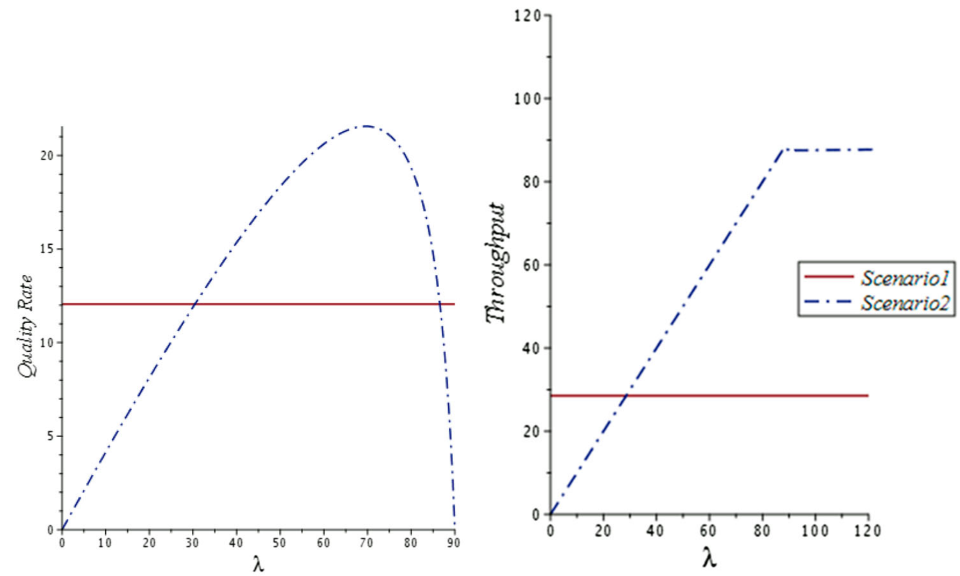
4. Comparing the two scenarios

In this section, we compare the two scenarios when in both cases B_i and D_i are exponentially distributed as defined above. Clearly $E[Q^{TJN}] < E[Q]$ for each individual product. Hence, a sufficient condition for the system's quality rate under scenario 2, $\lambda E[Q^{TJN}]$, to be lower than that under scenario 1, $E[Q]/E[T]$, is $\lambda < 1/E[T]$. Since the above is only a sufficient condition, we compare numerically the quality rates under the two scenarios.

4.1. Identical sites

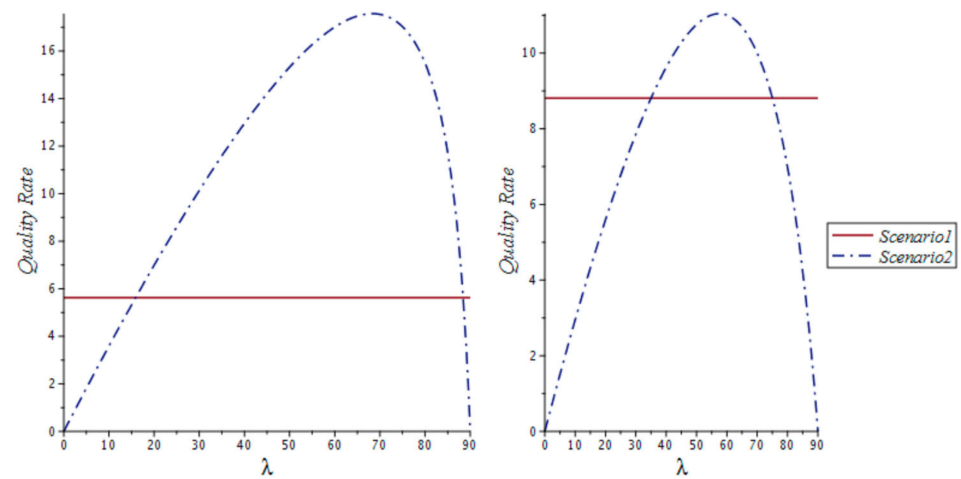
Assume that all processing stages are identical and all inspection stages are also identical, and compare quality rates as function of λ where $\delta_i = 1$. Let $s_i = 1$, $\mu_i = 90$, $\xi_i = 120$, $p_i = f_i = 0.1$ for $i = 1, \ldots, n-1$ and $f_n = 0.1$, $p_n = 0.9$. Steady state is achieved for scenario 2 when $\lambda < 90$. For a simple case where there are only two sites, $n = 2$, and the scaling factor is $\gamma = 5$, Figure 2 depicts the quality rates and the throughputs of the two scenario. Throughputs are equal when $\lambda = 28.5$. Quality rates are equal first when $\lambda^* = 30.43$ and then when $\lambda^{**} = 86.5$. When $\lambda < \lambda^*$, scenario 1 is better than scenario 2 since the former expected quality is higher while its throughput is higher or slightly lower than that of scenario 2. However, when $\lambda^* < \lambda < \lambda^{**}$ the production quality rate of scenario 2 is higher than that of scenario 1 since installing in-process buffers allows the throughput of scenario 2 to increase so that, although expected quality per product is lower under this scenario, the overhaul quality rate is higher than under scenario 1. When $\lambda > \lambda^{**}$ the TJN is too congested so that scenario 1 leads to a higher quality rate than that of scenario 2. That is, it is better to let each product traverse the network alone.

Figure 3 depicts how quality rates vary with incremental changes in the number of sites n and the scaling factor γ. Specifically, in Figure 3(a) $n = 5$ and in Figure 3(b) $\gamma = 15$, where all other parameters stay the same. As depicted in Figure 3(a), increasing the number of sites clearly does not affect the throughput of scenario 2 (as long as steady state is achieved), but reduces the expected quality of each single product. Under scenario 1, both the throughput $1/E[T]$ and the



a. Quality rate **b.** Throughput

Figure 2. Performance measures as a function of λ. (a) Quality rate. (b) Throughput.



a. $n = 5$ **b.** $\gamma = 15$

Figure 3. Quality rate as a function of λ. (a) $n = 5$. (b) $\gamma = 15$.

expected quality of a product $E[Q]$ are reduced as n increases, so that the production quality rate decreases. Increasing the scaling parameter γ reduces the quality rates of both scenarios since now the quality deteriorates faster as a function of the traversal time as depicted in Figure 3(b). For example, the quality rate in scenario 1 when $\gamma = 5$ is about 12 (see solid line in Figure 2(a)) while when $\gamma = 15$ it is close to 9.

4.2. Non-identical sites

Consider a network with two sites having identical inspection rates, $\xi_i = 120$, and different failure probabilities, $f_1 = 0.2$ and $f_2 = 0.8$. The processing times at each site are not identical with rates $\mu_1 = 30$ and $\mu_2 = 60$ at sites 1 and 2, respectively. We further assume that the product must pass through both sites and set $\delta_i = 1$. In this case, when sites are non-identical, the order at which the sites are arranged affects the quality rate of the production network. Note that when site 1 is the first in line the maximal rate so that steady state is achieved is $\lambda = 30\ (= \mu_1)$, whereas when site 2 is the first the maximal rate so that steady state is achieved is $\lambda = 60\ (= \mu_2)$, while the arrival rate to the following site is $(1 - f_2)\lambda = 12 < 30$.

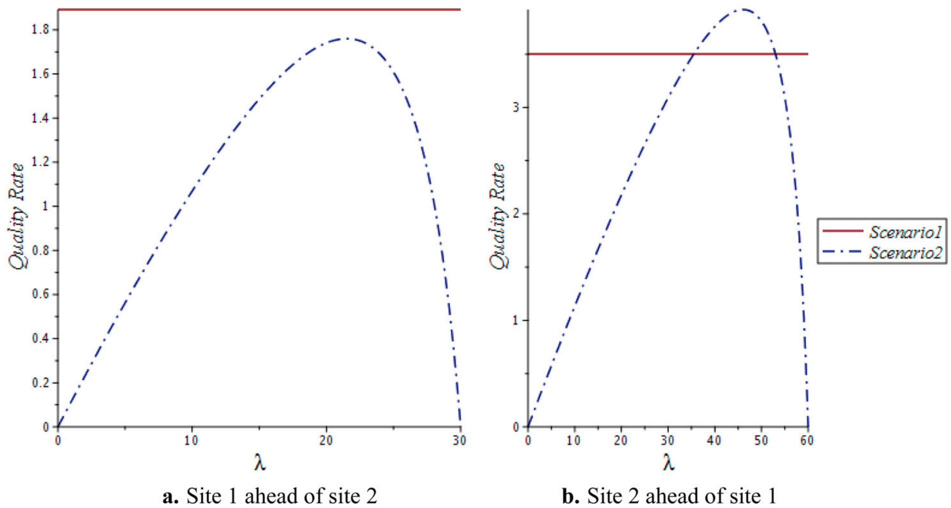


Figure 4. Quality rate as a function of λ. (a) Site 1 ahead of site 2. (b) Site 2 ahead of site 1.

As depicted in Figure 4(a), when site 1 is ahead of site 2, the production quality rate of scenario 1 is always higher than that under scenario 2. This result follows since increasing the arrival rate up to the steady-state limit is not enough to overcome the quality reduction. The quality rates when switching the order so that site 2 is first is depicted in Figure 4(b). In this case, since λ can be increased up to 60, the quality rate of scenario 2 excides that of scenario 1 when $35 < \lambda < 52$.

This numerical example demonstrates that the quality rate of the production line can be maximised in case a controller can order the sites and adjust the arrival rate to the network (i.e. the rate of inserting products to the production line).

5. Optimal order of sites

As discussed in the introduction, in flexible production environments the final product can be produced according to several different orders of operations. Practical examples of operations sequencing can be found in product assembly lines (e.g. Lee and Tang 1998; Righter and Shanthikumar 2001) and machining operations in the mechanical industry (e.g. Bard and Feo 1989; Cheung, Song, and Zhang 2017). Examples of flexible production environments in which deterioration takes place include process job shops in the chemical industry (Dennis and Meredith 2000; Shi, Yue, and Zhao 2014) and in the pharmaceutical industry (Dogan-Sahiner and Altiok 1998; Stefansson et al. 2011).

In this section, assuming that the order of the n sites can be determined by a controller, the objective is to find an optimal ordering policy that maximises the quality rate of the production network. It is further assumed that a product must pass through all n sites (i.e. $p_i = 0$ for $i = 1, 2, \ldots, n-1$).

Proposition 3 Under Scenario 1 the network production quality rate is maximised when the sites are arranged in a decreasing order of the index $f_i/E[R_i]$.

Proof Applying an interchange argument, it is sufficient to consider the case $n = 2$ and examine the order (1,2) versus the order (2,1). The claim is proved since the expected quality $E[Q]$ is identical under each order, whereas $E[T]_{1,2} \leq E[T]_{2,1}$ if and only if $f_1/E[R_1] \geq f_2/E[R_2]$. ■

Under scenario 2, there is no easy-to-implement index-type policy since the effective arrival rate to each site depends on the order of the sites, which itself affects the corresponding expected quality.

6. Conclusions

The common emphasis when studying and analysing n-site stochastic tandem production networks, such as Jackson-type systems, is on developing the joint probability distribution function of the site occupancies. Thus far, only a few studies of multistage production processes have addressed the issue of product deterioration during processing – and those that do consider it assume that such deterioration is linear over time.

In this paper we focus on the latter issue, assuming that product quality deteriorates exponentially over time. Two scenarios are investigated: (i) each product moves through the network on its own, such that a new product starts its processing sequence only after the preceding product has exited and (ii) the network is a tandem Jackson-type system, where each site is an unlimited-buffer M/M/1 queue. Analytical expressions for a product's expected sojourn time in the system and for its resulting expected quality are derived. Then, under the assumption that a product must pass through all sites and that these sites can be sequenced in any order, we derive easy to implement index-type ordering rules that maximise the expected quality rate of the production network. Our results can be utilised when designing similar production-type networks when quality issues are the leading concern. From a managerial standpoint, since tandem production systems involving product quality deterioration are highly prevalent in practice, our study provides an approach to evaluating and optimising the performance of such systems.

One way to extend this research is by allowing feedback from a site (machine) all the way back to the beginning of the network. That is, upon unsuccessful operation at any site, the product is fed back to the first site and the entire sequence of operations starts again. In addition, while it is assumed in this study that each site represents a reliable machine that does not break, a possible extension is to allow each machine to fail, and to consider the effect of down time and up time of each machine on the quality value of the final product. A further extension would be to investigate multi-server queues at each stage.

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